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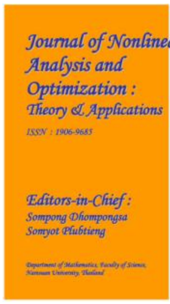

CHANNEL REPLACMENT MODEL UNDER TYPE-2 FUZZY SET

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1. INTRODUCTION:

“operation research is the systematic, method-oriented study of the basic structure, characteristics, function and relationships of an organization to provide the executive with a sound, scientific and quantitative basis for decision making” – E. L. Arnoff and M. J. Netzorg. Decision theory is an prominent fiels in operations Research. In day-to-day life, one has to take decision about minor or major issues and the decision has to be an optimal decision for the issue under consideration. There are three essential characteristics for any decision, namely, objective, alternative and influencing factor (constraints). Replacement theory is the study of finding which item to replace and whether or not to replace them and also if it has to be replaced, which replacement policy has to be preferred and also if it has to be replaced in group then when it has to be replaced in order to arrive at an optimal solution.
Information theory is the mathematical study of quantification, storage and communication of information from the source to the destination. The information (or) message which has to be sent from the source to the destination is transmitted through a communication channel. Unit hyper cube is an n-dimensional analogue of a square and a cube whose length is one. In this paper, the communication channel is considered as a unit hypercube. If there are a finite number of communication channels stops working for a period of time. Then it is necessary to check whether that particular channel needs to be replaced immediately or is it possible to transmit the information effectively through anyone communication channel available. If the information can be transmitted through any of the available communication channels, then it is necessary to decide which communication channel is optimum for transmitting the information effectively.
But the data regarding the efficiency of the other channels will not be precise and so fuzzy set theory, in particular. Type-2 fuzzy sets are used in order to identify the optimal communication channel among the other available channels. Thus, the channel which is not working for a short period of time could be replaced with an alternative optimal channel which was identified using type-2 fuzzy sets.

2. PRELIMARIES:

2.1 BASIC DEFINITIONS:

DEFINITION 2.1.1 – FUZZY SET
If R is a collection of objects denoted generically by r, then a fuzzy set
$\tilde{s}$ *in R is a set of ordered pair*:

$\tilde{s} = \{(r, \mu_{\tilde{s}}(r)) | r \in R\}$
Where $\mu_{\tilde{s}}(r)$ is called the membership function or grade of membership or degree of compatibility or degree of truth of r in $\tilde{s}$ that maps R to the membership space M.

DEFINITION 2.1.2-TYPE-2 FUZZY SET:
A type-2 fuzzy set is a fuzzy set where membership values are type-1 fuzzy sets on the closed interval [0,1].
DEFINITION 2.1.3-TYPE-M FUZZY SET:
A type-m fuzzy set is a fuzzy set whose membership values are type-m-1, m>1 fuzzy sets on[0,1].
DEFINITION 2.1.4-REPLACEMENT POLICY:
The replacement problem is to find when an (communication channel) item should be replaced by another (communication channel) item when it fails or when the maintance cost of the item is very high.
But when the communication channel is unavailable for a short period of time, then we have to check whether the current channel has to be replaced with an updated channel or alternative channel can be used for the transmission of information.
DEFINITION 2.1.5- TRANSMISSION MODE:
Transmission mode or communication mode means transferring data between two devices.
There are three different types of transmission modes, namely, simplex mode, half duplex mode and full duplex mode.
DEFINITION 2.1.5.1-SIMPLEX MODE:
When the communication is unidirectional (i.e) only one of the two devise on a link can send the information and the other device can only receive the information sent.
SENDER RECEIVER

Diagram 2.1.5.1

DEFINITION 2.1.6- METHODS OF DATA TRANSMISSION:
Data can be transmitted from one place to another place by using different methods. There are three major methods of data transmission, namely, unicast, multicast and broadcast.
DEFINITION 2.1.6.1- MULTICAST DATA TRANSMISSION:
Multicasting has one or more sender and multiple receivers while transferring information from one place to another.

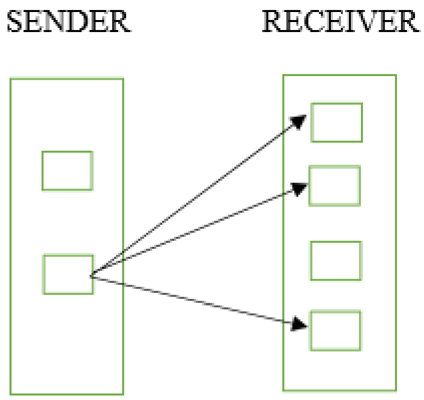

Diagram 2.1.6.1

2.2 DECISION THEORY:
A decision can be defined as the selection by the decision-maker if an act, considered to be the optimal solution according to some pre-designated standard, from among the several available choices. Thus, decision is made from among a set of well-defined alternative courses of action.
A decision maker has to in the position to answer the following three questions in order to arrive at the optimal solution for the problem under consideration: (i) what are the decision alternatives? (ii) what

are the restriction to take a decision? (iii) what is an appropriate objective criterion for evaluating the alternatives?
But in most of the real-life situations, the answer for these three questions would be uncertain.
Thus the decision has to be taken with 'partially uncertainty' or 'uncertainty'. Thus, this uncertainty in data can be effectively handled by fuzzy set theory.

2.3 REPLACEMENT MODELS:

Replacement is an integral part of every system and it deals with the problem of replacement of machined, equipment, vehicles, furniture, spare part, ect. In general, the replacement of the current equipment is necessary because of the following four reasons: replacement of equipment that fail suddenly, replacement of equipment that deteriorate gradually with respect to time, replacement of equipment that are outdated, replacement of equipment whose maintenance cost increase when the time (age) increases.

In this paper, a new replacement model is constructed. There are finite number of communication channels in a communication system, one particular channel is not working only for a particular short period of time in the communicate on system. Now, it has to be decided whether the current communication channel has to be replaced with a new updated communicate on channel or the information which has to be transmitted through the channel which may not be working only for a particular short duration of time can be transmitted properly through an alternative channel in this communication system only for the time period when that particular channel which is under consideration for replacement is unavailable. If no such alternative communication channel is available to transmit the information, then the channel has to be replaced with an updated version of communication channel. If there are alterative channels in the communication system which can successfully transmit the information then the decision maker has to choose the alternative channel which can optimally transmit the information.
For instance, consider a communication system with three communication channels, say, A, B, C: here, channel B is not working for a period of time.

A	B	C	$\bar{B}$	$A \cup C$	$\bar{B} \mapsto A \cup C$
0	0	0	1	0	0
0	0	1	1	1	1
0	1	0	0	0	0
1	0	0	1	1	1
0	1	1	0	1	0
1	0	1	1	1	1
1	1	0	0	1	0
1	1	1	0	1	0

Clearly, only when all the three channels are not able to transmit the information, the channel "B" has to be replaced. If the data about the working efficiency of all the communication channels under consideration are well-known and crisp, then the above tabular column could be used.
But in most if the real-life situations the data values would not be crisp and so fuzzy set theory has to be applied to obtain an optimal solution.

2.4 UNIT HYPERCUBE:

In geometry, a hypercube is defined as an n-dimensional analogue of a square and a cube. In particular, a unit hypercube is a hypercube whose side has length one unit. In fuzzy theory, the set of all fuzzy subsets equals to the unit hypercube, which is defined by In and is defined as
In = [0,1]n. Clearly, the vertices of the unit hypercube $is\ non\ fuzzy\ in\ nature$, $(i.\ e)$, $non-fuzzy\ sets$. $Also$, the midpoint of the unit hypercube In is maximally fuzzy as all the membership values of the midpoint of the unit hypercube In is exactly equal to ½. The unit hypercube In could be

sliced into $2n$ hyper-squares by extending the slides of the hyperplanes from the midpoint to the edges of the unit hypercube.

2.5 TRANSMISSION OF INFORMATION THROUGH COMMUNICATION CHANNELS:

Each and every channel in the communication system would be considered as an unit hypercube. Here a communication system is considered where a finite number of communication channels are arranged in a parallel manner. All the channels in the communication system under consideration use simplex mode of communication. When the information is transmitted from the source, multicast data transmission method is used to transmit the information successfully to the destination. To transmit the information from the source to the desired destination, there are five essential components of a communication system-sender, encoder, communication channel, decoder and receiver. Here, we consider a communication system where one among the finite number of communication channels would stop working only for a short period of time but resumes its job after that short period of time with good efficiency. Now, it is mandatory to check whether that communication channel has to be replaced with another updated communication channel or it is enough to find an alternative communication channel in that communication system in order to transmit the information effectively only for that time period when that channel is unavailable.

Also, the study of information theory is based upon the fundamental theorem which states that "It is possible to transmit information through a noisy channel at any rate less than the channel capacity with an arbitrarily small probability of error".

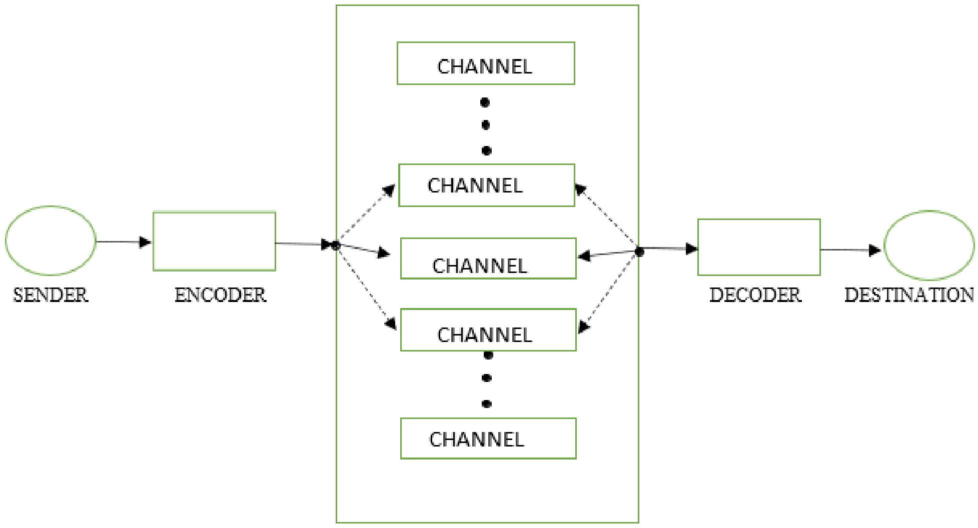

Diagram 2.5.1

2.6 APPLICATION:

This replacement model can be applied to the communication channels which would stop working only for a short duration but then resumes its work with good efficiency, for instance, (i) when the communication channel due to poor climate conditions, (ii) when the communication channel is under maintenance.

3. ILLUSTRATION:

There are three communication channels, namely, A, B, C which are placed in a parallel manner and the information is transmitted through simplex mode and also multicast information transmission method is followed. Due to unavoidable reasons, the communication channel "B" could not work for a particular time period. For the data given below, check whether it is necessary to replace the communication channel "B". if the channel "B" need not be replaced for the given data as the information transmission would be completed by channel "A" or channel "C", which channel could be optimal for information transmission if not channel "B".

$A = \{ (I_1/A, A/0.3), (I_2/A, A/0.5), (I_3/A, A/0.8)$
$(I_4/A, A/0.2), (I_5/A, A/0.4), (I_6/A, A/0.1),$
$(I_7/A, A/0.9), (I_8/A, A/0.2), (I_9/A, A/0.3)\}$

$C = \{ (I_1/C, C/0.6), (I_2/C, C/0.5), (I_3/C, C/0.2)$
$(I_4/C, C/0.8), (I_5/C, C/0.6), (I_6/C, C/0.9)$
$(I_7/C, C/0.1), (I_8/C, C/0.8), (I_9/C, C/0.7)\}$

SOLUTION:

For the first information transmission: $I_1/\mu_{I_1}(x)$

$I_1/\mu_{\tilde{A}}(x)+\mu_{\tilde{C}}(x) = I_1/0.3+0.6 = I_1/0.9$

For the second information transmission: $I_2/\mu_{I_2}(x)$

$I_2/\mu_{\tilde{A}}(x)+\mu_{\tilde{C}}(x) = I_2/0.5+0.5 = I_2/1.0$

For the third information transmission: $I_3/\mu_{I_3}(x)$

$I_3/\mu_{\tilde{A}}(x)+\mu_{\tilde{C}}(x) = I_3/0.8+0.2 = I_3/1.0$

For the fourth information transmission: $I_4/\mu_{I_4}(x)$

$I_4/\mu_{\tilde{A}}(x)+\mu_{\tilde{C}}(x) = I_4/0.2+0.8 = I_4/1.0$

For the fifth information transmission: $I_5/\mu_{I_5}(x)$

$I_5/\mu_{\tilde{A}}(x)+\mu_{\tilde{C}}(x) = I_5/0.4+0.6 = I_5/1.0$

For the sixth information transmission: $I_6/\mu_{I_6}(x)$

$I_6/\mu_{\tilde{A}}(x)+\mu_{\tilde{C}}(x) = I_6/0.1+0.9 = I_6/1.0$

For the seventh information transmission: $I_7/\mu_{I_7}(x)$

$I_7/\mu_{\tilde{A}}(x)+\mu_{\tilde{C}}(x) = I_7/0.9+0.1 = I_7/1.0$

For the eighth information transmission: $I_8/\mu_{I_8}(x)$

$I_8/\mu_{\tilde{A}}(x)+\mu_{\tilde{C}}(x) = I_8/0.2+0.8 = I_8/1.0$

For the ninth information transmission: $I_9/\mu_{I_9}(x)$

$I_9/\mu_{\tilde{A}}(x)+\mu_{\tilde{C}}(x) = I_9/0.3+0.7 = I_9/1.0$

Clearly, all the information transmission has the membership value 1.0 expect the first information which has the membership value 0.9. If the membership value of information transmission is 1.0, then the information is surely transmitted without much of error. Form the above data, there is only one information transmission which has membership function 0.9 otherwise the membership of the other information transmission is 1.0, which means on the whole, the information is transmitted either through the information communication channel A or through the information communication channel C in case the channel B is unavailable due to unavoidable reasons for a particular period of time. Thus, the communication channel "B" need not be replaced with an updated versions of the communication channel.

As the communication channel "B" is unavailable (i.e) not working for a particular period of time, the information which has to be transmitted by the communication channel "B" has to be transmitted either through the channel "A" or through the channel "C".

For the communication channel "A":

$E(I_{1A}) = [\mu_{1A}(x) * log\, \mu_{1A}(x)]$
$= 0.3 * \log 0.3$
$= 0.3 * [-0.5228]$
$\Rightarrow E(I_{1A}) =- 0.1568$
$E(I_{2A}) = [\mu_{2A}(x) * log\, \mu_{2A}(x)]$
$= 0.5 * \log 0.5$
$= 0.5 * [-0.3010]$
$\Rightarrow E(I_{2A}) =- 0.1505$
$E(I_{3A}) = [\mu_{3A}(x) * log\, \mu_{3A}(x)]$
$= 0.8 * [-0.0969]$
$\Rightarrow E(I_{3A}) =- 0.0775$
$E(I_{4A}) = [\mu_{4A}(x) * log\, \mu_{4A}(x)]$
$= 0.2 * [-0.6989]$
$\Rightarrow E(I_{4A}) =- 0.1397$
$E(I_{5A}) = [\mu_{5A}(x) * log\, \mu_{5A}(x)]$
$= 0.4 * [-0.3979]$
$\Rightarrow E(I_{5A}) =- 0.1591$
$E(I_{6A}) = [\mu_{6A}(x) * log\, \mu_{6A}(x)]$
$= 0.1 * [-1]$
$\Rightarrow E(I_{6A}) =- 0.1$
$E(I_{7A}) = [\mu_{7A}(x) * log\, \mu_{7A}(x)]$
$= 0.9 * [-0.045]$
$\Rightarrow E(I_{7A}) =- 0.0411$
$E(I_{8A}) = [\mu_{8A}(x) * log\, \mu_{8A}(x)]$
$= 0.2 * [-0.6989]$
$\Rightarrow E(I_{8A}) =- 0.1397$
$E(I_{9A}) = [\mu_{9A}(x) * log\, \mu_{9A}(x)]$
$= 0.3 * [-0.5228]$
$\Rightarrow E(I_{9A}) =- 0.1568$

$$\therefore E(I_A) = -\frac{[\sum E_{i=1}^{9} (I_{iA})]}{9}$$

$$\therefore E(I_A) = [E(I_{1A}) + E(I_{2A}) + E(I_{3A}) + \dots + E(I_{8A}) + E(I_{9A})]/9$$

$$=-\frac{[-0.1568-0.1505-0.0775-0.1397-0.1591-0.1-0.0411-0.1397-0.1568]}{9}$$

$$=\frac{1.1212}{9}$$

$E(I_A) = 0.1246$ ---(1)

Similarly, for the communication channel "C":

$E(I_{1C}) = [\mu_{1C}(x) * log\ \mu_{1C}(x)]$
$= 0.6 * [-0.2218]$
$\Rightarrow E(I_{1C}) =- 0.1331$
$E(I_{2C}) = [\mu_{2C}(x) * log\ \mu_{2C}(x)]$
$= 0.5 * [-0.3010]$
$\Rightarrow E(I_{2C}) =- 0.1505$

$E(I_{3C}) = [\mu_{3C}(x) * log\ \mu_{3C}(x)]$
$= 0.2 * [-0.6989]$
$\Rightarrow E(I_{3C}) =- 0.1397$
$E(I_{4C}) = [\mu_{4C}(x) * log\ \mu_{4A}(x)]$
$= 0.8 * [-0.0969]$
$\Rightarrow E(I_{4C}) =- 0.0775$
$E(I_{5C}) = [\mu_{5C}(x) * log\ \mu_{5C}(x)]$
$= 0.6 * [-0.2218]$
$\Rightarrow E(I_{5C}) =- 0.1331$
$E(I_{6C}) = [\mu_{6C}(x) * log\ \mu_{6C}(x)]$
$= 0.9 * [-0.0457]$
$\Rightarrow E(I_{6C}) =- 0.0411$
$E(I_{7C}) = [\mu_{7C}(x) * log\ \mu_{7C}(x)]$
$= 0.1 * [-1]$
$\Rightarrow E(I_{7C}) =- 0.1$
$E(I_{8C}) = [\mu_{8C}(x) * log\ \mu_{8}(x)]$
$= 0.8 * [-0.0969]$
$\Rightarrow E(I_{8C}) =- 0.0775$
$E(I_{9C}) = [\mu_{9C}(x) * log\ \mu_{9C}(x)]$
$= 0.7 * [-0.15449]$
$\Rightarrow E(I_{9C}) =- 0.1084$

$$\therefore E(I_C) = -\frac{[\sum E_{i=1}^{9} (I_{iC})]}{9}$$
$$=-\frac{[-0.1331-0.1505-0.1397-0.0775-0.1331-0.0411-0.0775-0.1084]}{9}$$
$$=\frac{0.9609}{9}$$
$$E(I_C) = 0.1068 \text{ --(2)}$$

From (1) and (2)
$E(I_A) = 0.1246 \geq 0.1068 = E(I_C)$

Hence, the entropy of the communication channel A and the entropy of the communication channel C are almost the same and so both the communication channels can transmit the information. But to be even more precise, the communication channel A is preferred in order to transmit the information optimally.

4. CONCLUSION:

Operations Research is simply the science of management. Decision theory plays a prominent role in Operations Research. In order to take the best decision in replacement of any equipment, in particular, communication channels the data about the communication channels has to be certain. But in most if the real life situations, the data regarding the communication channels are partially uncertain or uncertain. In order to deal with this kind of uncertainty, there is scope of application of fuzzy set theory in order to take optimal decision. Thus, the fuzzy phenomena could have better approximation of the real life phenomena. a different replacement model has been constructed which is illustrated with an example.

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